

Echo State and Band-pass Networks with aqueous memristors: leaky reservoir computing with a leaky substrate

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Recurrent Neural Networks (RNN) are extensively employed for processing sequential data such as time series. Reservoir computing (RC) has drawn attention as an RNN framework due to its fixed network that does not require training, making it an attractive for hardware based machine learning. We establish an explicit correspondence between the well-established mathematical RC implementations of Echo State Networks and Band-pass Networks with Leaky Integrator nodes on the one hand and a physical circuit containing iontronic simple volatile memristors on the other. These aqueous iontronic devices employ ion transport through water as signal carriers, and feature a voltage-dependent (memory) conductance. The activation function and the dynamics of the Leaky Integrator nodes naturally materialise as the (dynamic) conductance properties of iontronic memristors, while a simple fixed local current-to-voltage update rule at the memristor terminals facilitates the relevant matrix coupling between nodes. We process various time series, including pressure data from simulated airways during breathing that can be directly fed into the network due to the intrinsic responsiveness of iontronic devices to applied pressures. This is done while using established physical equations of motion of iontronic memristors for the internal dynamics of the circuit.

Reservoir computing (RC) is a proven method for processing temporal data and has drawn more recent attention as a suitable framework for hardware based machine learning. Echo State and Band-pass Networks are extensively studied implementations of RC. We propose a novel physical circuit design for hardware realisations of these networks based on fluidic iontronic memristors. Using the underlying equations of motion of these devices, we process several time series, including simulated respiratory pressure waveforms, exploiting iontronics' intrinsic sensitivity to applied pressures. Our derived one-to-one correspondence between the mathematical descriptions of these RC paradigms and the physics of emerging iontronic devices offers a blueprint for physically embedded temporal processing with an emerging substrate.

1 Introduction

Reservoir computing (RC) has gained significant attention as a Recurrent Neural Network paradigm for processing temporal data [1]. RC employs a fixed high-dimensional reservoir (i.e. a dynamical system with many internal states) whose dynamics are driven by input signals, with the benefit that only a simple readout function requires training for classification tasks. Although the rise of computational capacity for training in the past few years has somewhat mitigated this benefit, new attention has recently been drawn to RC for hardware-based implementations as the fixed nature of the reservoir circumvents complicated internal tuning of the RC circuit [2]. Recent research, for instance, has explored the use of physical systems such as electronic, electrochemical, optical, and mechanical devices [2, 3]. However, although the use of physical substrates is informed by the established mathematical frameworks for software RC, establishing a deeper physical equiv-

alence is challenging.

In this work, we establish an explicit one-to-one correspondence between the physical equations of iontronic memristors placed within a peripheral circuit on the one hand and the governing equations of Echo State Networks (ESNs) and Band-pass Networks (BPNs) with Leaky Integrator nodes (LI-ESNs and LI-BPNs, respectively) on the other. Iontronics exploit aqueous ionic and molecular transport, akin to the brain's medium, and can therefore provide striking similarities with the brain in neuromorphic computing implementations [4–8] such as RC [9]. Additionally, the easily tunable memory timescales of iontronic platforms [9–13] naturally match the relatively slow timescales found in natural or biological signals, something that is challenging within conventional fast solid-state devices [14]. Moreover, we show how the pressure dependence of such fluidic systems can provide a direct physical conversion from a biological pressure-based signal to the circuit input without any intervention or interaction required from outside the network. However, iontronic computing is still in its infancy and considerable developments are necessary to achieve iontronic computing systems [15]. In this work we *(i)* propose a circuit based on the emerging “leaky” substrate of aqueous iontronics that would be capable of advanced RC applications, *(ii)* propose a one-to-one correspondence between this physical (iontronic) circuit and the well-established mathematical LI-ESN and LI-BPN descriptions, *(iii)* leverage the unique property of slow easily tuneable memory timescales of iontronic memristors, and *(iv)* exploit the intrinsic pressure responsiveness of iontronics to directly convert pressure signal inputs on-chip. Due to the equivalence between our proposed physical device and the abstract mathematical framework, the (extensive) theoretical results previously derived for ESNs and BPNs [17–25] can be directly translated to the proposed physical circuit, without requiring us to reinvent the wheel for RC on a physical substrate. All code used for our results is available at [26]. Our results not

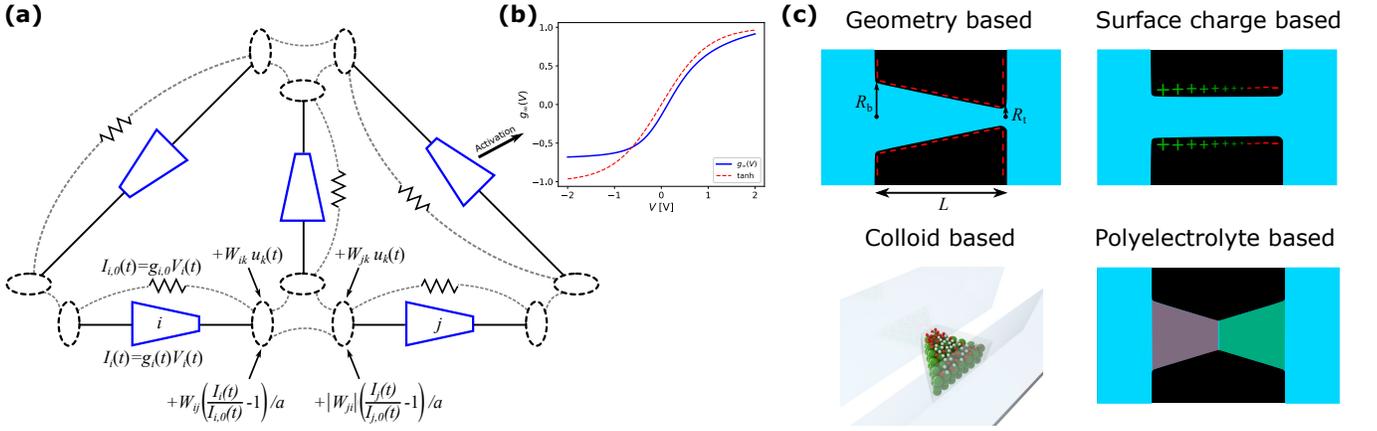


FIG. 1. **(a)** Schematic of the physical Leaky Integrator Echo State or Band-pass network circuit containing (iontronic) Simple Volatile Memristors. Resistors are connected in parallel to the memristors with conductances equal to the equilibrium conductance of their respective parallel memristors. The terminals at either end of the memristors convert the incoming currents of neighbouring memristor-resistor pairs to voltages of the next time step. For simplicity, only the inputs to and coupling between i and j memristors are included. **(b)** Several candidate iontronic Simple Volatile Memristors where the conductance memory is geometry-based (top left) [10, 11], surface charge-based (top-right) [13, 16], colloid-based [9] (bottom left), and polyelectrolyte-based (bottom right) [12]. All results in this work are from geometry-based iontronic memristors [11].

only advance the theoretical understanding of RC in physical systems but also provide a pathway for the development of new (iontronic) hardware-based RC implementations.

2 Echo State and Band-pass Networks

We consider LI-ESNs that converts a K -dimensional input $\mathbf{u}(t)$ with N reservoir neurons at state $\mathbf{x}(t)$ to an L -dimensional output $\mathbf{y}(t)$ at time t , here without output feedback and without direct input-to-output coupling. Such an LI-ESN is governed by [18]

$$\dot{\mathbf{x}} = \frac{1}{c} (-a\mathbf{x} + f(\mathbf{W}^{\text{in}}\mathbf{u} + \mathbf{W}\mathbf{x})), \quad (2.1)$$

$$\mathbf{y} = g(\mathbf{W}^{\text{out}}\mathbf{x}). \quad (2.2)$$

Here $c \in \mathbb{R}$ is a global relaxation time parameter, $a \in \mathbb{R}$ is the leaking rate, $\mathbf{u} = \mathbf{u}(t) \in \mathbb{R}^K$ is the input, $\mathbf{x} = \mathbf{x}(t) \in \mathbb{R}^N$ the state of the reservoir neurons, $f: \mathbb{R} \rightarrow \mathbb{R}$ is a sigmoidal activation function that is applied element-wise to its input, and $\mathbf{y} = \mathbf{y}(t) \in \mathbb{R}^L$ is the output vector determined by a function $g: \mathbb{R} \rightarrow \mathbb{R}$ that is applied element-wise to $\mathbf{W}^{\text{out}}\mathbf{x}$. Here we will set $f(x) = \tanh(x)$, a standard choice[18], and g will be the identity function such that $\mathbf{y} = \mathbf{W}^{\text{out}}\mathbf{x}$. The input and reservoir states are coupled through the input matrix $\mathbf{W}^{\text{in}} \in \mathbb{R}^{N \times K}$, the internal matrix $\mathbf{W} \in \mathbb{R}^{N \times N}$, and the output matrix $\mathbf{W}^{\text{out}} \in \mathbb{R}^{L \times N}$. Notably, only \mathbf{W}^{out} needs to be found through training, which we here perform via ridge regression[27], with the other matrices being initialised randomly. In its most general form there is also an output-feedback term $\mathbf{W}^{\text{fb}}\mathbf{y}$ in the argument of f , which is not of relevance for the input processing we focus on here [18]. Additionally, in general one can also directly couple the input to the output according to

$\mathbf{y} = g(\mathbf{W}^{\text{out}}[\mathbf{x}; \mathbf{u}])$ with $[\cdot]$ denoting vector concatenation (in this case $\mathbf{W}^{\text{out}} \in \mathbb{R}^{L \times (K+N)}$), which we also omit here to simplify the physical circuit realization.

The physical circuit we will introduce in Sec. 3.1 is in principle described by continuous equations as Eq. (2.1). However, inputs \mathbf{u} are often discrete-time sampled. Additionally, here we simulate the circuit using physical equations of the internally used iontronic memristors, but this will also require discretization of the underlying equations. Using Euler discretization with (time) stepsize δ and $t = n\delta$ with $n \in \mathbb{N}$, we see that Eq. (2.1) for $n \geq 0$ becomes[18]

$$\mathbf{x}(n+1) = \left(1 - \frac{a\delta}{c}\right) \mathbf{x}(n) + \frac{\delta}{c} f(\mathbf{W}^{\text{in}}\mathbf{u}(n\delta) + \mathbf{W}\mathbf{x}(n)). \quad (2.3)$$

2.1 Echo State property

A key stability property of ESNs is the *echo state property* (ESP), which is defined by[18]

Definition 1. An ESN with reservoir states $\mathbf{x}(n)$ has the echo state property if for any compact $C \subset \mathbb{R}^K$ and any two starting states $\mathbf{x}(0)$ and $\mathbf{x}'(0)$, there exists a sequence $(\delta_h)_{h=0,1,2,\dots}$ that converges to 0 such that for any input sequence $(\mathbf{u}(n))_{n=0,1,2,\dots} \subseteq C$ it holds that $\|\mathbf{x}(h) - \mathbf{x}'(h)\| \leq \delta_h$.

Heuristically, Def. 1 tells us that an ESN with the ESP “forgets” its initial state at a rate independent from the input sequence or the precise initial state.

There are various constraints that guarantee the echo state property in Leaky Integrator ESNs [18, 20]. The condition we will use here is that the spectral radius $\rho(\mathbf{M})$ of

$\mathbf{M} = (\delta/c)|\mathbf{W}| + (1 - a\delta/c)\mathbf{I}$ satisfies $\rho(\mathbf{M}) < 1$. A simple algorithm for constructing an internal weight matrix that guarantees the echo state property is given by [20]

1. Generate a random matrix \mathbf{W} with only non-negative elements $w_{ij} \geq 0$.
2. Rescale \mathbf{W} such that the spectral radius $\rho(\mathbf{M})$ of the matrix $\mathbf{M} = (\delta/c)\mathbf{W} + (1 - a\delta/c)\mathbf{I}$ satisfies $\rho(\mathbf{M}) < 1$.
3. Change the sign of a desired number of elements w_{ij} .

Internal weight matrices \mathbf{W} with spectral radii smaller than 1 often also display the ESP, but this is not a guarantee [20]. Alternatively, one could generate a random \mathbf{W} and then check if $\rho(\mathbf{M}) < 1$ afterwards. The available code[26] provides above-mentioned algorithm as an optional setting to guarantee the ESP. Lastly, $a\frac{\delta}{c} \leq 1$ is a natural constraint.

2.2 Band-pass network

A LI-BPN is similar to a LI-ESN as in Eqs. (2.1) and (2.2), with one key difference. Each node can individually be designed to be sensitive to certain frequencies, which we implement here by providing each node with its own characteristic relaxation timescale c_i [21], as opposed to a single global timescale c for the entire network. So the parameter $c \in \mathbb{R}$ is replaced by a vector $\mathbf{c} \in \mathbb{R}^N$ and Eq. (2.1) naturally becomes

$$\dot{\mathbf{x}} = (-a\mathbf{x} + f(\mathbf{W}^{\text{in}}\mathbf{u} + \mathbf{W}\mathbf{x})) \oslash \mathbf{c}, \quad (2.4)$$

with $\oslash \mathbf{c}$ element-wise (Hadamard) division.

3 Physical LI-ESN with iontronic memristors

Consider the circuit schematically drawn in Fig. 1(a) containing (iontronic) memristors (blue), which we will describe in more detail in Sec. 3.1.1. The voltages at the terminals (dashed ellipses) obey a fixed local current-to-voltage update rule, which we will describe in detail in Sec. 3.1.2. We will show in Sec. 3.2 that the physical circuit design in Fig. 1(a) is equivalent to the general mathematical LI-ESN description as in Eq. (2.1). Lastly, in Sec. 3.3 we will extend this equivalence to LI-BPNs as in Eq. (2.4).

3.1 Physical ESN circuit

3.1.1 (Iontronic) Simple Volatile Memristors

Memristors, characterised by their history-dependent conductance, have drawn major interest as fundamental devices for neuromorphic computing architectures [30]. Consequently, many different types of memristors with various conductance memory features exist [31–33]. Inspired by the brain’s aqueous medium and ionic signal carriers, iontronics that rely on ionic transport in an aqueous environment are

emerging as a substrate for neuromorphic computing implementations [4–8]. Of importance to this work is that various iontronic devices also feature a coupling between their electric properties and applied pressures [34–36], where e.g. an applied pressure can drive a so-called electric *streaming current* [37, 38].

The (iontronic) memristors schematically drawn in blue in Fig. 1(a) are *Simple Volatile Memristors* (SVMs) [39]. It has been demonstrated theoretically [11, 16] and experimentally [9, 10] that various fluidic iontronic memristors behave as SVMs [39], of which the electric conductance $g_i(t)$ is time-dependent and obeys the equation of motion (EOM)

$$\frac{dg_i}{dt} = \frac{g_{i,\infty}(V_i(t)) - g_i(t)}{\tau_i}. \quad (3.1)$$

Here $g_{i,\infty}(V_i)$ is the steady-state conductance for a given voltage V_i , which is typically a sigmoidal function around the equilibrium conductance $g_{i,0} = g_{i,\infty}(0)$. It has been theoretically derived [11, 16] and experimentally observed [9, 10, 12] that the intrinsic memory timescale τ_i of various iontronic SVMs scales quadratically with the device length L_i according to

$$\tau_i \propto \frac{L_i^2}{D}, \quad (3.2)$$

where D is the ionic diffusion coefficient, assumed equal for all ionic species of the aqueous electrolyte involved. Due to its dependence on L_i , the timescale τ_i can be individually chosen for each SVM across a wide range, which we will use in Sec. 3.3 to implement the individual relaxation times in LI-BPNs. A variety of different iontronic SVMs are candidates for the circuit we propose here, including channels where the conductance memory is geometry-based [10, 11], surface charge-based [16], colloid-based [9], and polyelectrolyte-based [12], as schematically depicted in Fig. 1(c).

For our network demonstrations here we chose to consider conical channel SVMs [11], but the results are representative of any SVM with a sigmoidal steady-state conductance. Specifically, we consider microfluidic channels as drawn in the top left of Fig. 1 with a base radius $R_b = 200$ nm, a tip radius $R_t = 50$ nm, a charge on the channel’s surface of $-2.4 \cdot 10^{-22}$ C/nm², filled with an aqueous 1:1 electrolyte with equilibrium ion concentrations of 0.1 mM for both the positive and negative ions. The conductance of the channel is voltage-dependent and shown in Fig. 1(b) in blue (normalised and centred around 0). The proportionality constant in Eq. (3.2) can vary between iontronic devices, but the channels [11] feature the relation $\tau_i = \frac{L_i^2}{12D}$, with lengths that can be fabricated from nm lengths [40, 41] all the way to mm length[42] scales, theoretically corresponding to a broad timescale range from $\sim 10^{-9}$ s up to $\sim 10^3$ s domains. While experimental evidence for the full range is still limited, the order $\sim 0.1 - 1$ s timescales that we will use here have been observed experimentally [9, 10, 43]. The lengths L_i of the channels vary between different network applications, and even within individual networks for BPNs, to implement the different timescales τ_i as per Eq. (3.2). The full detailed physics and remaining parameters are described in Appendix A.

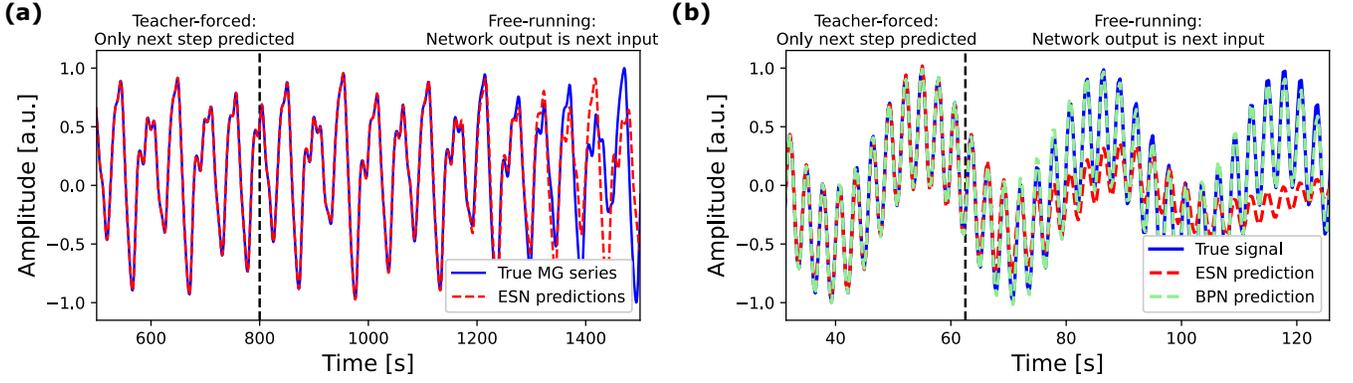


FIG. 2. **(a)** Mackey-Glass series predictions from a physical LI-ESN circuit containing iontronic conical channel memristors[11]. For $t < 800$, only the next step is predicted (with time stepsize of $\delta = 1$ s), for $t > 800$ the network receives no further input from the true time series and uses its own predicted output as input for the next step. Predicting 84 steps ahead yielded a RMSE of $\text{RMSE}_{84} \approx 0.001$, averaged over 20 network initializations, comparable to earlier results using LI-ESNs [19] and outperforming several neural network based approaches [28, 29]. **(b)** Harmonic time series predictions of an LI-ESN (red) and LI-BPN (green) containing 12 iontronic SVMs, showing that LI-ESNs struggle with the variations in signal timescales.

3.1.2 Current-to-voltage update rule

Consider the circuit schematically drawn in Fig. 1(a), where voltage terminals (dashed ellipses) connect parallel pairs of an Ohmic resistor and an SVM, here in the form of cone-shaped iontronic microfluidic channels [11] (blue). Memristors are two-terminal devices with $V_{i,t}$ and $V_{i,b}$ the voltages at the tip and base terminal, respectively, defined such that the voltage $V_i = V_{i,t} - V_{i,b}$ over the SVM increases the conductance for positive V_i . Between the terminal pairs two currents flow in parallel, a current $I_i = g_i(t)V_i(t)$ through the SVM and a current $I_{i,0} = g_{i,0}V_i(t)$ through the resistor with a fixed conductance $g_{i,0}$. The terminals obey the same update rule for the tip and base voltages $V_{i,t}$ and $V_{i,b}$, which depends on the currents I_j and $I_{j,0}$ of the neighboring terminals according to

$$V_{i,t} = \sum_{j:W_{ij}>0} W_{ij} \left(\frac{I_j}{I_{j,0}} - 1 \right) a^{-1} + \sum_{j:W_{ij}^{\text{in}}>0} W_{ij}^{\text{in}} u_j(t), \quad (3.3)$$

$$V_{i,b} = \sum_{j:W_{ij}<0} |W_{ij}| \left(\frac{I_j}{I_{j,0}} - 1 \right) a^{-1} + \sum_{j:W_{ij}^{\text{in}}<0} |W_{ij}^{\text{in}}| u_j(t). \quad (3.4)$$

Here W_{ij} , W_{ij}^{in} , and a are fixed and known *a priori*, $I_j(t)$ and $I_{0,j}(t)$ are physical currents that only need to be measured locally. Lastly, $u_j(t)$ is the dynamic input. Depending on the input type, the input can feature an additional scaling factor $s^{\text{in}} \in \mathbb{R}$ such that $u_j(t) = s^{\text{in}} \tilde{u}_j(t)$. This factor can fix the units and ensure the input stays within a reasonable $\sim \pm 1$ V voltage regime. Notably, in Sec. 4.3 $u_j(t)$ will receive its own local (pressure-to-)current-to-voltage update rule, where we analyze biological pressure signals as inputs by placing additional microfluidic channels between the pressure source and the SVM terminals. In such microfluidic channels, pressures are known to drive (electrical) *streaming currents* through the channels [37, 38], which can then be converted according to

a current-to-voltage update rule similar to Eq.(3.3), thereby providing a direct physical conversion between a biological signal and the ESN or BPN input without any intervention or interaction required from outside the network.

Although some functionality is assumed for the peripheral circuits at the terminals, this concerns only a straightforward conversion of locally measured currents to voltages with some a priori known fixed parameters. The use of peripheral circuitry for current-to-voltage conversions is relatively standard within neuromorphics, e.g. in the common neuromorphic circuits of coupled crossbar arrays that emulate artificial neural network current-to-voltage converters are employed to transform one array's current outputs to another array's voltage inputs [44].

3.2 Physical circuit and LI-ESN equivalence

For simplicity, let us initially consider all SVMs have equal length $L_i = L$ and therefore equal timescales $\tau_i = \tau$. In Sec. 3.3 we will make the straightforward extension to a range of timescales $\boldsymbol{\tau} \in \mathbb{R}^N$, equivalent to a Band-pass Network. We will show that the physical circuit design in Fig. 1(a) is equivalent to the general mathematical LI-ESN description as in Eq. (2.1). The circuit will thereby be endowed with all its relevant derived properties, capabilities, and understandings [17–24], while the actual dynamics emerge from the intrinsic physics of the circuit, rather than numerically solving Eq. (2.1) in software. Specifically, the conductance EOM Eq. (3.1) and steady state conductance $g_{i,\infty}(V_i)$ naturally assume the role of the ESN dynamics and the activation function. Furthermore, the relative polarity of the voltage depending on the orientation of the SVMs provides a natural method to encode either positive or negative (adjacency) weight elements.

We introduce a straightforward conversion to a dimensionless conductance $g_i(t)$ normalized by $g_{i,0} = g_{i,\infty}(0)$ the equi-

librium conductance

$$g_i(t) = \frac{g_i(t) - g_{i,0}}{ag_{i,0}}, \quad (3.5)$$

$$g_{i,\infty}(V_i) = \frac{g_{i,\infty}(V_i) - g_{i,0}}{g_{i,0}} \approx \tanh(V_i). \quad (3.6)$$

We stress that all results presented in this work exclusively use the physical function $g_{i,\infty}(V_i)$ [11] for the activation function $g_{i,\infty}(V_i)$, shown in blue in Fig. 1(b), alongside the function $\tanh(V_i)$ (red). The abovementioned similarity $g_{i,\infty}(V_i) \approx \tanh(V_i)$ only serves to support the equivalence to LI-ESNs.

The EOM of $g_i(t)$ is straightforwardly found through Eq. (3.1) as follows

$$\begin{aligned} \frac{dg_i}{dt} &= \frac{1}{ag_{i,0}} \frac{(g_{i,\infty}(V_i) - g_{i,0}) - (g_i(t) - g_{i,0})}{\tau} \\ &= \frac{g_{i,\infty}(V_i) - ag_i(t)}{a\tau}. \end{aligned}$$

In vector-notation this becomes

$$\frac{d\mathbf{g}}{dt} = \frac{g_\infty(\mathbf{V}) - a\mathbf{g}(t)}{a\tau}, \quad (3.7)$$

where $g_\infty(\mathbf{V})$ is applied element-wise to \mathbf{V} .

Memristors are two terminal devices with voltages $V_{i,t}$ and $V_{i,b}$ at either terminal, respectively, here defined such that the voltage $V_i = V_{i,t} - V_{i,b}$ over the SVM increases the conductance for positive voltages. Both terminals have identical voltage update rules, which we now show are coupled to g as follows

$$\begin{aligned} V_{i,t} &= \sum_{j:W_{ij}>0} W_{ij} \left(\frac{I_j}{I_{j,0}} - 1 \right) a^{-1} + \sum_{j:W_{ij}^{\text{in}}>0} W_{ij}^{\text{in}} u_j(t) \\ &= \sum_{j:W_{ij}>0} W_{ij} \frac{g_j(t)V_j - g_{j,0}V_j}{ag_{j,0}V_j} + \sum_{j:W_{ij}^{\text{in}}>0} W_{ij}^{\text{in}} u_j(t) \\ &= \sum_{j:W_{ij}>0} W_{ij} g_j + \sum_{j:W_{ij}^{\text{in}}>0} W_{ij}^{\text{in}} u_j. \end{aligned}$$

Similarly

$$V_{i,b} = \sum_{j:W_{ij}<0} |W_{ij}| g_j(t) + \sum_{j:W_{ij}^{\text{in}}<0} |W_{ij}^{\text{in}}| u_j(t).$$

Therefore, the voltage over the SVM is given by

$$\begin{aligned} V_i &= V_{i,t} - V_{i,b} \\ &= \sum_{j:W_{ij}>0} W_{ij} g_j - \sum_{j:W_{ij}<0} |W_{ij}| g_j \\ &\quad + \sum_{j:W_{ij}^{\text{in}}>0} W_{ij}^{\text{in}} u_j - \sum_{j:W_{ij}^{\text{in}}<0} |W_{ij}^{\text{in}}| u_j, \end{aligned} \quad (3.8)$$

where we now see that negative weights are naturally encoded through the voltage sign reversal. Moreover, we see that the current-to-voltage conversion rule in Eq. (3.3) is equivalent to a matrix multiplication with the dimensionless conductances $g_i(t)$.

Compactly, Eq. (3.8) can be written in matrix vector notation as

$$\mathbf{V} = \mathbf{W}\mathbf{g} + \mathbf{W}^{\text{in}}\mathbf{u}, \quad (3.9)$$

such that \mathbf{g} evolves according to

$$\frac{d\mathbf{g}}{dt} = \frac{g_\infty(\mathbf{W}^{\text{in}}\mathbf{u} + \mathbf{W}\mathbf{g}) - a\mathbf{g}(t)}{a\tau} \quad (3.10)$$

which we recognize as (the arguments inside) the activation function f in Eq. (2.1). Therefore, the straightforward current-to-voltage update rule described in Eq. (3.3) facilitates the matrix coupling between the nodes. This does require that the voltages can be adjusted quasi-instantaneously compared to the timescale τ of the SVM. To complete the equivalence to Eq. (2.1), let us consider the identifications

$$\begin{aligned} g_\infty(x) &\approx \tanh(x) \leftrightarrow f(x) \\ \mathbf{g}(t) &\leftrightarrow \mathbf{x}(t) \\ a\tau &\leftrightarrow c. \end{aligned}$$

We now see that Eq. (3.10) is identical to Eq. (2.1), while being completely physically facilitated in the circuit shown in Fig. 1(a).

3.3 Physical LI-BPN

Because each SVM can straightforwardly be designed to feature its own timescale $\tau_i \propto L_i^2/D$ by varying the length L_i of the individual devices, we can easily go beyond ESNs to BPNs, which are known to perform considerably better on input tasks that feature components that span multiple frequencies [25].

Varying the lengths between the different SVMs corresponds to converting τ to a vector $\tau \rightarrow \boldsymbol{\tau} \in \mathbb{R}^N$ such that Eq. (3.10) becomes

$$\frac{d\mathbf{g}}{dt} = \frac{g_\infty(\mathbf{W}^{\text{in}}\mathbf{u} + \mathbf{W}\mathbf{g}) - a\mathbf{g}(t)}{a} \odot \boldsymbol{\tau}, \quad (3.11)$$

with $\odot \boldsymbol{\tau}$ element-wise (Hadamard) division. We note that the dimensionless $g_{i,\infty}(V)$ is independent from L_i , so the extension from Eq. (3.10) remains valid. With the same identification steps as in Sec. 3.2, we see that Eq. (3.11) is equivalent to the mathematical description of LI-BPNs as in Eq. (2.4). Therefore the circuit depicted in Fig. 1(a) can be designed to be either an ESN or BPN, depending on whether the channel lengths vary.

4 Timeseries analysis tasks

4.1 Mackey-Glass time series

To reproduce some of the known capabilities of LI-ESNs, and to compare to the time series prediction performance of

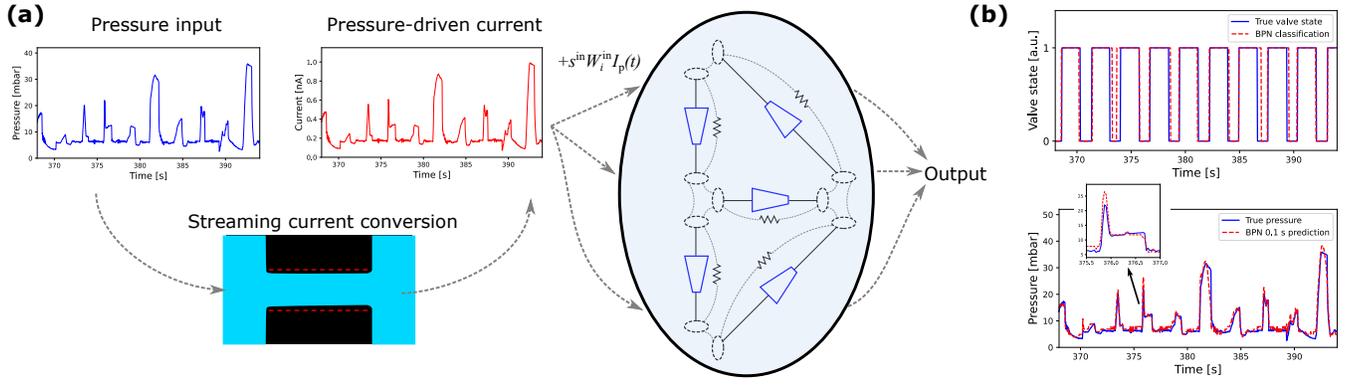


FIG. 3. **(a)** Schematic depiction of how the applied (airway) pressure (blue graph) drives an electric streaming current (red graph) through a cylindrical microfluidic channel (bottom) [37, 38]. This ionic current is then converted to voltage updates in the terminals of the iontronic SVMs, similar to how the ionic currents through the memristors are coupled to the neighbouring nodes. **(b)** Results of a classification task (top graph) of whether the expiratory valve (i.e. the valve that lets air out) is open or closed, using a small LI-BPN consisting of 8 iontronic SVMs. Additionally, a prediction task (bottom graph) of what the pressure will be 0.1 s in the future using a LI-BPN containing 200 iontronic SVMs. The inset shows that <0.1 s variations in the pressure are predicted by the network.

other methods, we use our iontronic SVM based circuit to predict the synthetic Mackey-Glass [45] time series $P(t)$. This time series is one of the most common generated datasets to test ESNs on [46], generated by

$$\frac{dP(t)}{dt} = \frac{\beta P(t - t_{\text{delay}})}{\theta + P(t - t_{\text{delay}})^n} - \gamma P(t), \quad (4.1)$$

where we use $\beta = 0.2$, $\theta = 1$, $\gamma = 0.1$, $n = 10$, and $t_{\text{delay}} = 17$. The first 17 time steps are randomly generated values in the range $[-1, 1)$. For the aforementioned parameters, Eq. (4.1) is known to feature a chaotic attractor [47]. A reservoir was used with parameters inspired by Ref. [18] of $K = 1$, $N = 400$, and $L = 1$, a network sparsity of 0.75, $c = 2.27$ s, $\delta = 1$ s, a leaking rate of $a = 0.9$, and an input scaling of $s^{\text{in}} = 0.5$ V. The output matrix \mathbf{W}^{out} was trained on a test set of length 21,000 s using ridge regression [27]. We distinguish between teacher-forced classification, where only the next time step needs to be predicted with the true signal as input, and free-running classification, where the network receives its own output as input for the next time step while receiving no information from the true signal. The resulting teacher-forced ($t < 800$ s) and free-running ($t > 800$ s) predictions are shown in Fig. 2(a).

In Fig. 2(a) we see that the network with $N = 400$ is able to accurately predict the Mackey-Glass series for several 100 s. To quantify this performance and to compare it to previous results, we calculate the root mean squared error (RMSE) of the prediction $\hat{P}_i(t_{\text{free}} + 84)$ of the true value $P_i(t_{\text{free}} + 84)$ 84 steps (i.e. 84 s) from the start t_{free} of the free-running mode for $T = 20$ different random initializations of the LI-ESN, given by $\text{RMSE}_{84} = \sum_{i=1}^{T=20} \sqrt{(\hat{P}_i(t_{\text{free}} + 84) - P_i(t_{\text{free}} + 84))^2} / T$. With our fully physically realisable circuit we find $\text{RMSE}_{84} \approx 0.001$. This outperforms several neural network based approaches [28, 29] and is comparable to earlier results using LI-ESNs [19]. This in and of itself is not surprising, as LI-ESNs with 400 nodes have long been shown to be capable of this [19]. Therefore, recreating this performance using the physi-

cal conical channel SVM equations supports our claim of this work that our *physical* circuit is equivalent to these ESNs and thereby to all their capabilities. We do note that performances of $\text{RMSE}_{84} < 0.1$ were still achieved for smaller networks of 100 SVM nodes, though variability between initializations increases for smaller networks.

Notably, in both predictions shown in Fig. 2 we use the physical steady-state conductance $g_{\infty}(V)$ as activation function [11]. Moreover, we are able to translate the parameters a and c to the physical length of the memristors, which for the results of Fig. 2(a), with diffusion coefficient $D = 1 \mu\text{m}^2\text{ms}^{-1}$, would be $L = 174 \mu\text{m}$. Otherwise, the remaining parameters (e.g. channel radius, salt concentration, surface charge, etc.) are as briefly reported in Sec. 3.1.1 and in full detail in Appendix A.

4.2 Band-pass network for multi-frequency signals

All nodes in LI-ESNs feature the same universal relaxation timescale c . Therefore, LI-ESNs can struggle with inputs that incorporate signal timescales of different magnitudes [25]. To demonstrate this, and how this can be overcome with iontronic SVMs, we consider a small LI-ESN of $K = 1$, $N = 12$, and $L = 1$, a network sparsity of 0.75, time step $\delta = \pi/10$ s, a timescale of $c = 1$ s, a leaking rate of $a = 0.9$, and an input scaling of $s^{\text{in}} = 0.1$ V. We use this network to predict a simple harmonic function that features oscillations on a ~ 1 s and ~ 10 s scale given by

$$y_{\text{har}}(t) = \sin(t) \cos(1.2t).$$

The resulting ESN predictions can be seen in the red curve of Fig. 2(b), where it is clear that the LI-ESN cannot reliably predict the time series in free-running form. Since we only model 12 SVM nodes with weights that are randomly generated, there is significant variability between different initial-

tions. However, the LI-ESN consistently performed poorly on correctly predicting the time series, as we will quantify below.

Converting our LI-ESN to a LI-BPN significantly improves the performance. In this case, timescales are drawn from a normal distribution $C_i \sim N(\mu_c, \sigma_c^2)$, with $\mu_c = 2$ and $\sigma_c = 3$, such that the timescale of each SVM node is set by $c_i = \max(\mu_c/5, C_i)$ to ensure all timescales are positive. More sophisticated methods of choosing the timescales can be considered, but for now this simple approach is sufficient to demonstrate the benefits that LI-BPNs provide. As shown in the green curve of Fig. 2(b), the resulting LI-BPN can successfully predict the time series with only 12 nodes. Again, there is some variability between different initiations of the small network, but the improved performance was consistent. The RMSE of predicting 55 s ahead from the start of the free-running mode, averaged over 100 different ESN and BPN initiations, shows that the LI-BPN features a 3 times lower RMSE. Quantitatively this seems like a somewhat marginal difference, but this is indicative of the qualitative observation that the higher frequency oscillations are often not captured by the ESN circuit. We stress that physically, this conversion from a ESN to BPN is straightforward, as the variability in timescales can be realized on-chip through varying the channel lengths.

4.3 Airway pressure as direct physical input

Thus far we have provided some demonstrations of analysing some synthetic time series with LI-ESNs and LI-BPNs containing iontronic SVMs. Here we will consider measurements of ventilator pressures that were designed to accurately mimic the airway pressures present in lungs during breathing [48]. For this task we will leverage two useful properties of iontronic SVMs, (i) their tuneable timescales that coincide with timescales of natural or biological origin, and (ii) the intrinsic responsiveness of iontronic systems to pressure inputs.

As schematically shown in the blue curve of Fig. 3(a), the input pressure is applied at one end of a cylindrical microfluidic channel Fig. 3(a, bottom) of length $L = 200 \mu\text{m}$ and radius $R = 25 \mu\text{m}$, carrying a typical surface potential of $\psi_0 = -40 \text{ mV}$. These channels feature a coupling between pressure and electric (ionic) current, specifically the resulting pressure drop $\Delta p(t)$ drives a so-called electric *streaming current* (red curve of Fig. 3(a)) given by[38]

$$I_p(t) = \pi R^2 \frac{\varepsilon \psi_0}{\eta} \frac{\Delta p(t)}{L},$$

with η and ε the shear viscosity and electric permittivity of water. The length L and radius R of the channel are chosen such that for the typical biological pressure signal amplitude of $\sim 10 \text{ mbar}$, the streaming current $I_p(t)$ will be of order $\sim 1 \text{ nA}$. This is a current strength that can be reliably measured for single channels driven by pressure [34, 37]. The ionic streaming current becomes the input $u(t) = s^{\text{in}} I_p(t)$. Similar to how the ionic currents through the SVMs are converted to

voltage contribution updates within the terminals, the streaming current can now be directly coupled to the SVMs such that the voltage contribution $s^{\text{in}} I_p(t) W_i^{\text{in}}$ of the inputs is as per Eq. (3.3).

Using the ventilator pressure data[48], an example of which is shown (blue) in Fig. 3(a), we perform two tasks, the easier task of classifying whether the expiratory valve (i.e. the valve that lets air out) is open or closed, and the harder task of predicting the next 0.1 s of the pressure profile. In the top graph of Fig. 3(b) we show the valve classifications in red, compared to the true value in blue, corresponding to the input depicted in Fig. 3(a). For these classifications a small LI-BPN circuit was modelled with only $N = 8$ iontronic SVMs. We chose this number of SVM nodes such that the network reliably (i.e. averaged over 20 initialisations) classifies correctly $> 90\%$ of the time. To reliably achieve the same $> 90\%$ correctness with a LI-ESN circuit of similar parameters, at least 28 nodes were needed, again demonstrating how performance can be significantly improved by varying the lengths of the fluidic memristors inside the circuit. The LI-BPN featured a network sparsity of 0.75, time step $\delta = 0.034035 \text{ s}$ (matching with the data), timescale parameters $\mu_c = 0.15 \text{ s}$ and $\sigma_c = 0.15 \text{ s}$, input scaling $s^{\text{in}} = 3.7 \text{ V/nA}$, and a leaking rate of $a = 0.66$. In the bottom graph of Fig. 3(b) we show the pressure predictions using a larger LI-BPN circuit containing 200 SVMs, a network sparsity of 0.95, time step $\delta = 0.034035 \text{ s}$, timescale parameters $\mu_c = 0.1 \text{ s}$ and $\sigma_c = 0.25 \text{ s}$, and a leaking rate of $a = 0.66$. Each point in the red graph is a prediction of 0.1 s ahead, achieving an RMSE of $\approx 3 \text{ mbar}$ (measured over the full test length). Although most individual pressure waves are longer than this 0.1 s timescale, there are certainly $< 0.1 \text{ s}$ features within each pressure wave that are still correctly predicted by the network, as shown in the inset. We note that this task is especially difficult because normally more parameters accompany each individual waveform [48], whereas here we solely provide the pressure as input.

5 Discussion and conclusion

In summary, we proposed a physical circuit design that exhibits an explicit one-to-one correspondence to the well-established mathematical description of the extensively studied reservoir computing frameworks of Leaky Integrator Echo State and Band-pass Networks[17–25]. This circuit contains fluidic iontronic memristors [4–8], whose voltage dependent conductance and conductance memory facilitate the activation function and dynamics of the nodes, respectively. The terminals at either end of the memristors feature fixed peripheral circuits that convert locally measured currents to voltages, shown here to form a physical realisation of the matrix coupling in LI-ESNs and LI-BPNs. By solving the physical equations of microfluidic conical channel memristors [11] inside such a circuit, we reproduced earlier demonstrated performance on predictions of the Mackey-Glass time series[19], supporting our claim that such a circuit is a physical manifestation of LI-ESNs. A desirable property that these iontronic memristors possess the dependence of their conduc-

tance memory timescale on the channel length, i.e. each memristor can individually be designed to feature a certain relaxation time. This corresponds to going from a LI-ESN to a LI-BPN, which are known to perform significantly better on inputs that feature multiple timescales [25]. This is demonstrated here by showing that a circuit with 12 channels of differing lengths, i.e. a physical LI-BPN circuit, performs significantly better on predicting a harmonic time series with features across different timescales than physical LI-ESN circuits where all channels are of the same length. Lastly, we leverage another unique property of microfluidic devices by exploiting their intrinsic responsiveness to applied pressures. Applied pressures can drive electric currents through microfluidic channels, so a pressure signal can be converted to an ionic current through additional microfluidic channels between the input source and the network. By coupling this to the existing current-to-voltage conversion in the terminals, the pressure is converted to the network's input without any intervention or interaction required from outside the network. By solving the physical equations that quantify this pressure driven current, we were able to classify and predict features of (simulated) biophysically realistic data of airway pressures during breathing [48].

Some functionality of our proposed circuit design lies in the peripheral circuitry that connect the terminals of the memristors, namely converting currents of neighbouring memristors to updated voltages. Current-to-voltage converting peripheral circuits are common within neuromorphics [44], supporting that such circuitry can efficiently be implemented. We ignored device noise in this study, assuming that the currents through the memristors can be accurately and reliably measured. Interestingly, LI-ESNs have also been theoretically studied with noise [17, 18, 24], so both from a theoretical and from a physical implementation perspective this is an interesting direction to study next. Additionally, for future studies, analysing more real-world data, especially from biological (pressure) sources, would be a relevant expansion. Optimising a circuit design for one such specific task is of interest as here we focused mostly on establishing the correspondence between our physical circuit design, and LI-ESNs and LI-BPNs, but focused little on fine-tuning a specific circuit design.

In conclusion, our proposed iontronic memristor based physical circuit design is theoretically equivalent to the well established reservoir computing methods of Leaky Integrator Echo State and Band-pass Networks. This is supported by performing several time series prediction and classification tasks. The fluidic devices do not necessarily need to be integrated in a fully fluidic circuit, thereby circumventing existing challenges of manufacturing integrated fully fluidic chips [49]. The analysed time series notably include airway pressure signals as inputs, leveraging iontronic's intrinsic pressure responsiveness and natural timescales that match with biology, with pressure classification tasks being achieved with as few as 8 memristors. This work provides a design that can advance the field of iontronic (neuromorphic) computing, while exploiting some of iontronics' intrinsic properties.

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A Conical channel conductance

This Appendix is a summary of the results described in Ref.[11]. We consider an azimuthally symmetric single conical channel, schematically depicted in Fig. 1(b, top left), of length L (given for each specific network in the main text) with the central axis at radial coordinate $r = 0$ and a radius described by $R(x) = R_b - x\Delta R/L$ for $x \in [0, L]$ where $R_b = 200$ nm is the base radius at $x = 0$ and $R_t = R_b - \Delta R = 50$ nm the tip radius at $x = L \gg R_b$. The channel connects two bulk reservoirs of an incompressible aqueous 1:1 electrolyte with viscosity $\eta = 1.01$ mPa · s, mass density $\rho_m = 10^3$ kg · m⁻³ and electric permittivity $\epsilon = 0.71$ nF · m⁻¹, at the far side of both reservoirs we impose a fixed pressure $P = P_0$ and fixed ion concentrations $\rho_{\pm} = \rho_b = 0.1$ mM. The channel wall carries a uniform surface charge density $e\sigma = -0.0015$ enm⁻², screened by an electric double layer with Debye length $\lambda_D \approx 10$ nm, resulting in an electric surface potential of $\psi_0 \approx -10$ mV. The ions have concentrations $\rho_{\pm}(x, r)$, diffusion coefficients $D_{\pm} = D = 1$ $\mu\text{m}^2\text{ms}^{-1}$, and charge $\pm e$ with e the proton charge. Over the channel we impose an electric potential $V(t)$, defined as the voltage $V_t(t)$ in the tip reservoir minus the voltage $V_b(t)$ in the base reservoir.

The steady-state conductance of a conical channel depends on the voltage-dependent radially averaged salt concentration profile $\bar{\rho}_s(x, V) = 2 \int_0^{R(x)} r(\rho_+(x, r) + \rho_-(x, r))dr/R(x)^2$ that exhibits salt concentration polarisation upon an applied voltage. The consequent voltage-dependent steady-state channel conductance is described by [35]

$$\begin{aligned} \frac{g_{\infty}(V)}{g_0} &= \int_0^L \bar{\rho}_s(x, V) dx / (2\rho_b L) \\ &= 1 + \Delta g \int_0^L \left[\frac{x}{L} \frac{R_t}{R(x)} - \frac{e^{\text{Pe}(V)\frac{x}{L}\frac{R_t}{R_b R(x)} - 1}}{e^{\text{Pe}(V)\frac{R_t}{R_b} - 1}} \right] dx / L, \end{aligned} \quad (\text{A.1})$$

where $g_0 = (\pi R_t R_b / L)(2\rho_b e^2 D / k_B T)$, $\text{Pe}(V) = Q(V)L / (D\pi R_t^2)$ the Péclet number at the narrow end, $Q(V) = -\pi R_t R_b \epsilon \psi_0 / (\eta L)V$ the volumetric fluid flow through the channel, and $\Delta g \equiv -e\Delta R \eta \sigma D / (\rho_b R_b R_t \epsilon \psi_0 k_B T)$.

The dynamic (dimensional) conductance $g(t)$ is well described by [11]

$$\frac{dg(t)}{dt} = \frac{g_{\infty}(V(t)) - g(t)}{\tau}, \quad (\text{A.2})$$

with $\tau = L^2 / 12D$ the typical conductance memory time of the channel.

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